8/27 Recitation

In this recitation, we will use the commands found in the **Vectors** and **Proofs** notebooks; these can be found on canvas.

Open a new Jupyter notebook and label it something like **Recitation8.27**. Do all of your Julia calculations today in that notebook; be sure to save your work so you can reference it later!

1. Let
$$\mathbf{v}_1 = \begin{bmatrix} -1 \\ .5 \\ 0 \\ 0 \end{bmatrix}, \mathbf{v}_2 = \begin{bmatrix} 2 \\ 0 \\ -1 \\ 3 \end{bmatrix}, \mathbf{v}_3 = \begin{bmatrix} 9 \\ 7 \\ 3 \\ -2 \end{bmatrix}$$
Use Julia to perform the following calculations
   (a) Determine the linear combination of $\mathbf{v}_1, \mathbf{v}_2$ and $\mathbf{v}_3$ with weights $2, -3, \frac{5}{2}$.

   (b) Determine the dot product $\mathbf{v}_1 \cdot \mathbf{v}_2$.

2. Recall the distributive property: $c(\mathbf{v} + \mathbf{w}) = c\mathbf{v} + c\mathbf{w}$ for all vectors $\mathbf{v}, \mathbf{w} \in \mathbb{R}^n$ and scalars $c \in \mathbb{R}$.
   (a) Use Julia to verify this property for two symbolic vectors in $\mathbb{R}^4$.
   (b) Pick two concrete vectors $\mathbf{v}$ and $\mathbf{w}$ in $\mathbb{R}^2$ and a concrete scalar c. Verify the property is true geometrically for these choices; that is, use Julia to plot both $c(\mathbf{v} + \mathbf{w})$ and $c\mathbf{v} + c\mathbf{w}$ and compare.
   (c) Prove the distributive property in general, by hand.

Looking Ahead

3. Let $\mathbf{v} = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$. Sketch all multiples of $\mathbf{v}$ in the coordinate plane. The set of all such vectors is called the **span** of $\mathbf{v}$.

4. Let $\mathbf{v} = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$, $\mathbf{w} = \begin{bmatrix} 3 \\ 5 \end{bmatrix}$, and $\mathbf{b} = \begin{bmatrix} 2 \\ 6 \end{bmatrix}$. Express $\mathbf{b}$ as a linear combination of $\mathbf{v}$ and $\mathbf{w}$ in two ways:
   (a) Use Brute-force. Find two scalars c_1 and c_2 such that $c_1\mathbf{v} + c_2\mathbf{w} = \mathbf{b}$. You can do this by hand or use Julia to compute $c_1\mathbf{v} + c_2\mathbf{w}$ for different input scalars.
   (b) Develop a generalizable method*. Translate this linear combination equation into a system of two equations that you can solve systematically using algebra.

5. Open the **Matrix** notebook to see the commands you will be using next time. Play around with them.

*We will soon develop an algorithmic way to solve such problems